

# Comment on “General Relativity Resolves Galactic Rotation Without Exotic Dark Matter” by F.I. Cooperstock & S. Tieu

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## Abstract

The general relativistic model of Cooperstock & Tieu, which attempts to fit rotation curves of spiral galaxies without invoking dark matter, is tested empirically using observations of the Milky Way. In particular, predictions for the mass density in the solar neighbourhood and the vertical density distribution at the position of the Sun are compared with observations. It is shown that the model of Cooperstock & Tieu, which was so constructed that it gives an excellent fit of the observed rotation curve, singularly fails to reproduce the observed local mass density and the vertical density profile of the Milky Way.

*Key words:* galaxies: kinematics and dynamics, galaxies: dark matter

*PACS:* 98.35.Ce, 98.35.Df, 98.62.Dm, 98.62.Ck

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## 1 Introduction

Recently Cooperstock & Tieu [1] (hereafter CT05) have proposed a new approach to the interpretation of rotation curves of spiral galaxies, which is based on the theory of general relativity. They argue that even in the case of such weak gravitational fields as in galaxies certain non-linear terms in Einstein’s field equations play an important albeit hitherto neglected role. Their formalism is applied to concrete examples, and CT05 provide quantitative fits of

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the rotation curves of the Milky Way and three further external spiral galaxies and they derive mass models for these galaxies. The resulting models are quite flattened and their total masses are typically one order of magnitude lower than those of current models of spiral galaxies. In these models the flat outer rotation curves are usually modelled by massive dark halos. The low total masses estimated by CT05 can be accounted for by the baryonic mass content of the galaxies alone. CT05 conclude that it is thus not necessary to invoke “exotic dark matter” to model galactic rotation curves.

Although not yet in print, this spectacular result raised considerable interest but was also met with scepticism in the astronomical community. For instance CT05 have not dealt with the dark matter problem of galaxy clusters. A conceptual problem arises from the non continuously differentiable shapes of the density cusps of the vertical density profiles of the models at the galactic midplanes. This seems to indicate that each galaxy would at least formally harbour at its midplane a sheet of negative mass density [2], [3]. Other formal inconsistencies are discussed in [4]. In a rebuttal to these criticisms CT05 [5] maintain the claim of their original paper.

In this *comment* we demonstrate how observations of the Milky Way can be used as an *empirical* counter example against CT05’s conjecture of the dynamics of galactic disks.

## 2 The mass density in the solar neighbourhood and the vertical mass density profile of the Milky Way at the position of the Sun

According to CT05’s formalism the distribution of mass in their galaxy models is given by

$$\rho(r, z) = 8.36 \cdot 10^5 \left( \left( \sum_{n=1}^{10} k_n^2 C_n e^{-k_n |z|} J_0(k_n r) \right)^2 + \left( \sum_{n=1}^{10} k_n^2 C_n e^{-k_n |z|} J_1(k_n r) \right)^2 \right) \frac{M_\odot}{pc^3}, \quad (1)$$

where  $J_{0,1}$  denote Bessel functions of the first kind. The coefficients  $k_n$  and  $C_n$  have been determined by CT05 by fitting the corresponding model rotation curve to the observed rotation curve of the Milky Way and are given in their Table 1. Fig. 1 shows in the left panel the vertical mass density profile at the position of the Sun,  $\rho(r_\odot, z)$ , calculated with Eq. (1). The Sun lies close to the Galactic midplane,  $z \approx 0$ , and the galactocentric distance of the Sun is about  $r_\odot = 8$  kpc [6], but other determinations are discussed in the literature as well. Thus density profiles assuming  $r_\odot = 7$  kpc and  $r_\odot = 8.5$  kpc, which bracket

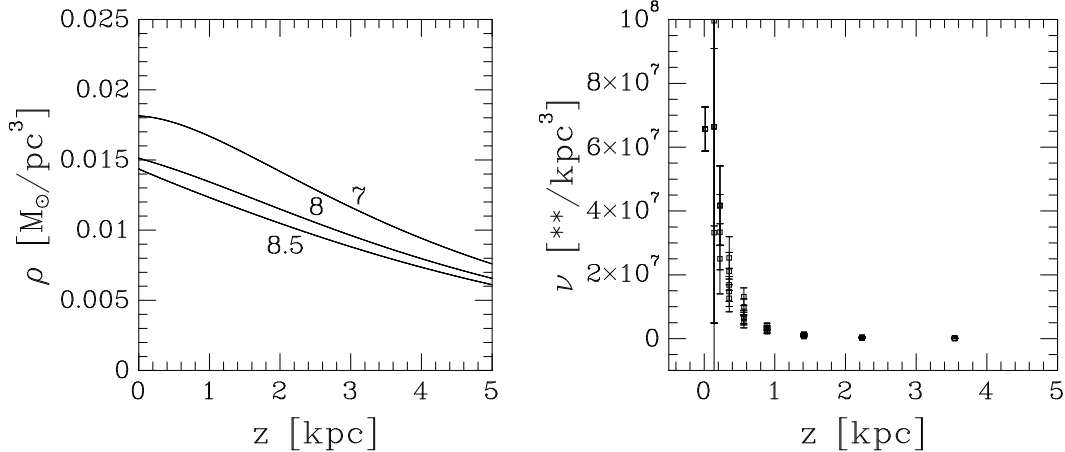


Fig. 1. Predicted versus observed vertical distribution of the mass density in the Milky Way at the position of the Sun. Left panel: Vertical distribution predicted by the mass model of Cooperstock & Tieu. The profiles are labelled by the assumed galactocentric distance of the Sun ranging from 7 to 8.5 kpc. Right panel: The observed distribution of stars perpendicular to the Galactic midplane.

the literature values for  $r_{\odot}$ , are also shown in Fig. 1. Holmberg & Flynn [7] have meticulously compiled an inventory of the contributions by the various phases of the interstellar gas and the stellar populations to the mass budget in the vicinity of the Sun and find a local mass density of  $\rho(r_{\odot}, 0) = 0.094 M_{\odot}/\text{pc}^3 = 6.3 \cdot 10^{-21} \text{ kg/m}^3$ . As described in [7] this value is consistent with dynamical measurements of the local mass density, if the gravitational force field is calculated in Newtonian approximation. However, as can be seen from Fig. 1 the mass model of CT05 predicts at the position of the Sun a density of about  $\rho(r_{\odot}, 0) = 0.015 M_{\odot}/\text{pc}^3 = 1.0 \cdot 10^{-21} \text{ kg/m}^3$ . This amounts to only 16 percent of the mass density actually observed in the form of baryons in the solar neighbourhood.

Moreover, the predicted shape of the vertical density distribution looks totally different from what is actually observed. In the right panel of Fig. 1 the observed number density distribution of stars perpendicular to the Galactic midplane at the position of the Sun,  $\nu(r_{\odot}, z)$ , is shown. The number densities have been determined with counts of K and M stars in five fields of the Calar Alto Deep Imaging Survey [8]. Since the CADIS star counts suffer from severe Poisson errors near to the midplane due to the conical counting volumes (cf. Fig. 1), the local normalization has been determined by counting stars of the same spectral types in the Fourth Catalogue of Nearby Stars [8], [9]. The CADIS fields point towards different galactic longitudes and latitudes so that the scatter of the data points in the right panel of Fig. 1 reflects also some mild variations of the vertical shape of the Galactic disk seen in the various direction. We may add that the vertical density profile derived from CADIS data is in perfect agreement with the results of Zheng et al. [10]. Early type stars and most of the interstellar gas are distributed in a narrow layer at the Galactic

midplane so that the overall distribution of baryons is even more concentrated towards the midplane than the late type stars, whereas the vertical distribution predicted by CT05's model is extremely shallow compared to the observations. Indeed the implied surface density of the disk at the position of the Sun is  $179 M_{\odot}/\text{pc}^2 = 0.37 \text{ kg/m}^2$ . Although the midplane density is much too low, the predicted surface density is a factor of about four higher than the observed surface density of baryons of  $48 M_{\odot}/\text{pc}^2 = 0.1 \text{ kg/m}^2$  [7]. As can be seen from Eq. (1) and Eq. (18) of CT05 any attempt to rescale the model by increasing the coefficients  $k_n$  in order to obtain a smaller scale height would alter also the radial shape of the predicted rotation curve  $V(r, z = 0)$  and thus destroy the fit to the observed rotation curve.

This implies that the model of CT05 for the Milky Way, which was so constructed that it gives an excellent fit of the observed rotation curve, has singularly failed to reproduce the independent observations of the local Galactic mass density and its vertical distribution. This one counter example casts, in our view, severe doubts on the viability of Cooperstock & Tieu's theory of the dynamics of galactic disks in general.

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